(G) In a central force field, prove that the angular momentum of a particle is conserved. 1½

(H) If the conservative force F is given by $F = -\frac{k}{r^2}$, then find the potential V. 1½

TKN/KS/16/5854

Bachelor of Science (B.Sc.) Semester—IV (C.B.S.) Examination

MATHEMATICS

(Mechanics)

Paper—II

Time : Three Hours] [Maximum Marks : 60

N.B. :— (1) Solve all the **FIVE** questions.

(2) All questions carry equal marks.

(3) Questions **1** to **4** have an alternative. Solve each question in full or its alternative in full.

UNIT—I

1. (A) Forces P_1, P_2, P_3, P_4, P_5, P_6 act along the sides of a regular hexagon taken in order. Show that they will be in equillibrium if $P_1 + P_2 + P_3 + P_4 + P_5 + P_6 = 0$ and $P_1 - P_4 = P_3 - P_6 = P_5 - P_2$. 6

(B) Four equal heavy uniform rods are freely joined so as to form a rhombus which is freely suspended by one angular point and the middle points of the two upper rods are connected by a light rod so that the rhombus can not collapse. Prove that the tension of this rod is 4W tan α, where W is the weight of each rod and 2α is the angle of the rhombus at the point of suspension. 6

OR

executes S.H.M. of period $\frac{2\pi}{n}$ and amplitude $\sqrt{a^2+b^2}$. 6

(D) A small bead P can slide on a smooth wire AB, being acted upon by a force per unit mass equal to μ/CP^2, where C is outside AB. Show that the time of small oscillation about its position of equilibrium is $\left(\frac{2\pi}{\sqrt{\mu}}\right)b^{3/2}$, where b is the perpendicular distance of C from AB. 6

UNIT—III

3. (A) If the virtual work of the forces of constraint vanishes for a mechanical system of particles, then prove that

$$\sum_i \left[\vec{F}_i^{(a)} - \dot{\vec{P}}_i\right] \circ \delta\vec{r}_i = 0, \quad i = 1,2, \ldots\ldots, n.$$

where $\vec{F}_i^{(a)} \equiv$ the applied force on i^{th} particle

$-\dot{\vec{p}}_i \equiv$ the reversed effective force on i^{th} particle,

$\delta\vec{r}_i \equiv$ the virtual displacement of i^{th} particle. 6

(B) Show that $F_r = m\ddot{r} - mr\,\dot{\theta}^2$ and $F_\theta = mr\ddot{\theta} + 2m\dot{r}\,\dot{\theta}$ from the motion of a particle by using plane polar coordinates r and θ. 6

OR

(C) Prove that the Lagrange's equations can be written as :

$$\frac{d}{dt}\left[\frac{\partial L}{\partial \dot{q}_j}\right] - \frac{\partial L}{\partial q_j} + \frac{\partial R}{\partial \dot{q}_j} = 0, \; j = 1, 2, \ldots., n,$$

where R is Rayleigh's dissipation function. 6

(D) Derive the Lagrange's equations of motion for a partly conservative system in the form

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_j}\right) - \frac{\partial L}{\partial q_j} = Q_j', \; j = 1, 2, \ldots\ldots, n,$$ where L refers to the conservative part and Q_j' refers to the forces which are not conservative. 6

UNIT—IV

4. (A) If the force law is f(r) in a central force field, then prove that a differential equation for central orbit is given by $\frac{\ell^2 u^2}{m}\left[\frac{d^2u}{d\theta^2} + u\right] = -f(1/u)$, where $\frac{1}{u} = r$. 6

(B) Show that if a particle describes a circular orbit under the influence of an attractive central force directed toward a point on the circle, then the force varies as the inverse fifth power of the distance. 6

OR

(C) A uniform chain has a horizontal span of 96 feet (ft.) and tension T at the upper end is twice that at the lowest. Show that the length of the chain is $\frac{96\sqrt{3}}{\log\left(2+\sqrt{3}\right)}$ ft. 6

(D) A uniform chain of length 'ℓ', is to be suspended from two points A and B, in the same horizontal line so that either terminal tension is 'n' times that at the lowest point. Show that the span AB must be

$$\frac{\ell}{\sqrt{n^2-1}}\log\left[n+\sqrt{n^2-1}\right].$$ 6

UNIT—II

2. (A) A particle describes the curve $r = ae^{m\theta}$ with a constant velocity. Find the components of velocity and acceleration along the radius vector and perpendicular to it. 6

(B) If the curve is an equiangular spiral $r = ae^{\theta\cot\alpha}$ and if the radius vector to the particle has constant angular velocity, show that the resultant accelaration of the particle makes an angle '2α' with the radius vector and is of magnitude ν^2/r, when ν is the speed of the particle. 6

OR

(C) The position of a particle moving in a straight line is given by $x = a \cos nt + b \sin nt$. Prove that it

(C) Prove that the problem of motion of two masses interacting only with each other can always be reduced to a problem of motion of a single mass. 6

(D) If the potential energy is a homogeneous function of degree –1 in the radius vector $\bar{r}_i$, then prove that the motion of a conservative system takes place in a finite region of space only if the total energy is negative. 6

UNIT—V

5. (A) Define : Like and Unlike parallel forces. Also write their resultant $\bar{R}$. 1½

(B) For a common catenary, prove the relation $y^2 = c^2 + s^2$. 1½

(C) Prove the relation : $\frac{d\hat{r}}{dt}=\frac{d\theta}{dt}\hat{n}$, where $\hat{r}$ and $\hat{n}$ are unit vectors along and perpendicular to radial direction respectively in XY-plane and θ is the angle made by the radius vector with an axis OX. 1½

(D) Prove that the acceleration of a point moving in a plane curve with uniform speed is $\rho\dot{\psi}^2$. 1½

(E) Define holonomic and nonholonomic constraints. 1½

(F) Find the equation of motion of a particle by using Newton's second law of motion. 1½